USE OF CERTAIN APPROXIMATIONS OF THE FRESNEL FORMULAS TO DETERMINE THE OPTICAL CONSTANTS OF ABSORBING MATERIALS

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The possibility of using Fresnel formula approximations to compute the angular dependence of optical properties and to determine the optical constants of opaque materials is discussed.

Knowledge of the optical constants n and k, the real and imaginary parts of the complex index of refraction, their variance, and temperature dependence is necessary both for the investigation of the optical and radiation properties of materials [1] and for the study of their microcharacteristics [2]. Hence, the development of methods to determine the optical constants, which permit extension of the spectral and temperature ranges of the measurements and increase the confidence in the data obtained, is of great interest.

The majority of methods to determine n and k are based on using a dependence between the measured parameters and the optical constants resulting from Fresnel formulas. The applicability of each specific method depends on the state and temperature of the material surface and is determined by the sensitivity of the parameter being measured to the magnitude of the optical constants.

As is known, the Fresnel formulas are derived from the Maxwell equations upon compliance with a number of conditions, one of which is the requirement for the presence of a plane interface between two media on the path of electromagnetic radiation propagation. The boundary conditions taking account of the inhomogeneity in the interface are hence not considered [3]. Therefore, the requirement for the presence of "ideal" specularity of the surface, which is essential in determining the optical constants of engineering materials which always have a rough surface, does not follow from the fact of using the Fresnel formulas. Compliance with this requirement is necessary in those cases when the spectral specular reflectivity, which is quite responsive to the magnitude of the roughness, is measured to determine the optical constants. This responsiveness is due to the fact that the geometry of the reflected light beam, identical for optically smooth surfaces to the incident beam geometry, varies as the optical magnitude of the roughness increases because of the spatial redistribution of the reflected light which causes a diminution in the specular reflectivity [4]. At the same time, the absorptivity α_λ or emissivity ε_λ as well as the reflectivity ρ_λ of the surface, taking account of the light reflected in a hemisphere above the specimen, remain invariant [5] while the optical magnitude of the roughness does not achieve a specific value.

Therefore, the measurement of α_λ, ρ_λ and ε_λ permits obtaining more accurate data to determine the optical constants than the measurement of the specular reflectivity does, since even a well-polished surface has some roughness [5].

According to the Fresnel equations and the Kirchhoff law, the interrelation between the quantities named and the optical constants for a given angle θ (illumination or radiation) is expressed by the formulas

$$\varepsilon_{\lambda,\theta}^s = \alpha_{\lambda,\theta}^s = 1 - \rho_{\lambda,\theta}^s = 1 - [(a - \cos\theta)^2 + b^2]/[(a + \cos\theta)^2 + b^2], \tag{1}$$

$$\varepsilon_{\lambda,\theta}^p = \alpha_{\lambda,\theta}^p = 1 - \rho_{\lambda,\theta}^p = 1 - \frac{[(a - \cos\theta)^2 + b^2][(a - \sin\theta\,\mathrm{tg}\,\theta)^2 + b^2]}{[(a + \cos\theta)^2 + b^2][(a + \sin\theta\,\mathrm{tg}\,\theta)^2 + b^2]}, \tag{2}$$

$$\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta} = 1 - \rho_{\lambda,\theta} = (\varepsilon_{\lambda,\theta}^p + \varepsilon_{\lambda,\theta}^s)/2, \tag{3}$$

where

$$a^2 = 1/2\{n^2 - k^2 - \sin^2\theta) + [(n^2 - k^2 - \sin^2\theta)^2 + 4n^2k^2]^{1/2}\};$$

$$b^2 = 1/2\{-(n^2 - k^2 - \sin^2\theta) + [(n^2 - k^2 - \sin^2\theta)^2 + 4n^2k^2]^{1/2}\}.$$

TABLE 1. Error in the Approximation $\Delta D^{*}_{\lambda,\theta}$, % (I) and Values of the Coefficients E/F (II) in (9)

θ, deg	Quantity	n	$\varepsilon_{\lambda,n}$					
			0,05	0,10	0,20	0,30	0,40	0,50
75	I	1	−0,062	−0,045	0,11	0,37	0,70	1,10
	II		1,5/7,8	1,5/4,7	1,5/3,1	1,5/2,5	1,5/2,3	1,5/2,1
	I	2	0,012	0,007	−0,046	−0,13	−0,22	−0,32
	II		1,9/14	1,9/8,3	1,9/5,1	1,9/4,0	1,9/3,5	1,9/3,1
	I	4	0,065	0,078	−0,021	−0,24	−0,57	−1,0
	II		2,8/28	2,8/15	2,8/8,9	2,7/6,8	2,7/5,7	2,7/5,0
	I	6	0,085	0,11	0,010	−0,24	−0,64	—
	II		3,7/41	3,6/22	3,6/13	3,5/9,4	3,5/7,8	—
84	I	1	−0,18	0,12	0,98	1,97	3,04	4,02
	II		1,2/2,1	1,2/1,6	1,2/1,4	1,2/1,4	1,2/1,3	1,2/1,3
	I	2	−0,17	−0,12	0,15	0,52	0,97	1,53
	II		1,4/3,2	1,4/2,3	1,4/1,8	1,4/1,7	1,4/1,6	1,4/1,6
	I	4	−0,075	−0,098	−0,098	−0,064	−0,05	0,080
	II		1,8/5,4	1,8/3,6	1,8/2,7	1,8/2,4	1,8/2,3	1,8/2,2
	I	6	−0,023	−0,051	−0,11	−0,18	−0,26	—
	II		2,2/7,6	2,2/4,9	2,2/3,5	2,2/3,0	2,2/2,8	—
	I	10	0,027	0,15	−0,074	−0,21	—	—
	II		3,0/12	3,0/7,5	3,0/5,2	2,9/4,4	—	—

This interrelation is simplified greatly for normal values of the quantities under consideration

$$\varepsilon_{\lambda,n} = \alpha_{\lambda,n} = 1 - \rho_{\lambda,n} = 4n/[(n+1)^2 + k^2]. \tag{4}$$

The solution of (1)–(3) for n and k by means of the results of measuring $\rho^{P}_{\lambda,\theta}$, $\rho^{S}_{\lambda,\theta}$ and θ is impossible without using an electronic computer [6], hence, a number of authors attempted to simplify the computations by using simpler analytical expressions which are a modification of the Fresnel formulas for the given parameters to be measured. In [7], for instance, the minimum value of the ratio $R_{min} = \rho^{P}_{\lambda,\theta}/\rho^{S}_{\lambda,\theta}$ and its corresponding angle θ (the second Brewster angle [8]) are used as such parameters, while the parameters being measured in [9] were $\varepsilon_{\lambda,n}$ and $\varepsilon_{\lambda,\theta}$ for θ = 45°. However, the relative simplicity in the computations achieved in this manner does not eliminate the constraints on the applicability of these methods because of the complexity of the relationship between n and k and the parameters being measured, and of the actually achievable measurement accuracy.

Methods to determine the optical constants of metals, based on measurements of the values of $\varepsilon^{S}_{\lambda,\theta}$ and $\varepsilon^{P}_{\lambda,\theta}$ or their corresponding brightnesses of the natural surface radiation $L^{S}_{\lambda,\theta}$ and $L^{P}_{\lambda,\theta}$ [10–13] for several angles θ relative to the normal to the surface have been developed especially rapidly recently. The polarization relationship $\beta_{\theta} = \varepsilon^{P}_{\lambda,\theta}/\varepsilon^{S}_{\lambda,\theta} = L^{P}_{\lambda,\theta}/L^{S}_{\lambda,\theta}$ is found from the measurement results for each angle θ and by solving the system of equations with the quantities β_{θ} and θ as parameters, the numerical values of n and k are determined. The disadvantage of these methods is the boundedness of the spectral and temperature ranges of their application due to the need to measure $\varepsilon^{S}_{\lambda,\theta}$. It is easy to see that it is expedient to perform measurements for large values of the angle θ in order to increase the resolution of the methods. Since the quantity $\varepsilon^{S}_{\lambda,\theta}$ (and its corresponding $L^{S}_{\lambda,\theta}$) decreases rapidly with the increase in θ, a high radiation level is necessary for its accurate measurement. This level is assured only at high temperatures in the ultraviolet and visible ranges of the spectrum, where metals have a sufficiently high emissivity, which indeed governs the low temperature limit of the measurements, or ∿1200°K (∿900°C) [12]. Another disadvantage is the complexity of the measurements needed to realize the methods [13].

Methods to determine the optical constants of opaque materials, including metals, are proposed in this paper which will permit overcoming the disadvantages noted to a significant extent, other conditions being equal.
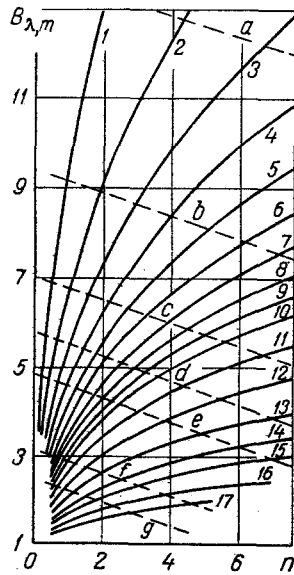
Fig. 1. Diagram of the dependence $B_{\lambda,m} = (L^P_{\lambda,\theta}/L^P_{\lambda,n})_{max} = (\varepsilon^P_{\lambda,\theta}/\varepsilon_{\lambda,n})_{max} = f(n, \varepsilon_{\lambda,\theta})$: $\varepsilon_{\lambda,n} = 0.01$ (1); 0.02 (2); 0.03 (3); 0.04 (4); 0.05 (5); 0.06 (6); 0.07 (7); 0.08 (8); 0.09 (9); 0.10 (10); 0.12 (11); 0.15 (12); 0.20 (13); 0.25 (14); 0.30 (15); 0.40 (16); 0.50 (17). $\theta_m = 88°$ (a); 87° (b); 86° (c); 85° (d); 84° (e); 80° (f); 77° (g).

An analysis of the results of a computation performed by means of (1)-(4) for a broad range of values of $\varepsilon_{\lambda,\theta}$, n and θ permitted finding a number of expressions to approximate the interrelation between these quantities, which can underlie methods to determine the optical constants.

It is established that the quantity $A_{\lambda,\theta}$ for metals can be computed for the angles $\theta \geq 60°$ by means of the approximation

$$A_{\lambda,\theta} = [1 + (0.4440 - 0.259)\,\varepsilon_{\lambda,n}\,(1.05 + \varepsilon_{\lambda,n})]\cos\theta, \tag{5}$$

where θ is the emission angle in radians.

Here and henceforth, the error in the approximation was determined by means of the expression

$$\Delta G^* = [(G^* - G)/G]\cdot 100\%. \tag{6}$$

For $\varepsilon_{\lambda,n} \leq 0.15$ and $n \leq 2.0$, which are characteristic for metals in the IR spectrum band, $\Delta A^*_{\lambda,\theta} \leq \pm 0.5\%$ in the mentioned angle range. The relative error in $A_{\lambda,\theta}$ due to the error in measurement of $\varepsilon_{\lambda,n}$ is determined by the ratio $\Delta A_{\lambda,n}/A_{\lambda,n} = q(\Delta\varepsilon_{\lambda,n}/\varepsilon_{\lambda,n})$, where $q \leq 0.08$ for $\varepsilon_{\lambda,n} \leq 0.15$ and $\theta \leq 85°$.

Therefore, by measuring the spectral normal emissivity of metals and the angular dependence of the relative spectral brightness of their surface $L_{\lambda,\theta}/L_{\lambda,n} = \varepsilon_{\lambda,\theta}/\varepsilon_{\lambda,n} = f(\theta)$ by using known methods [14], the values of $\varepsilon^S_{\lambda,\theta}$ and $\varepsilon^P_{\lambda,\theta}$ which permit evaluation of the values of the optical constants can be determined by using (3) and (5) for a given angle θ. Other conditions being equal, the measurements of $\varepsilon_{\lambda,n}$ and the relative angular dependence $f(\theta)$ are evidently simpler and more exact than the measurements of $\varepsilon^S_{\lambda,\theta}$ and $\varepsilon^P_{\lambda,\theta}$.

The disadvantage of the approximation (5) is that for $\varepsilon_{\lambda,n} = 0.05$ the condition $\Delta A^*_{\lambda,\theta} \leq \pm 0.5\%$ is valid for $n \geq 1$ while for $\varepsilon_{\lambda,n} = 0.35$ this condition is satisfied only for $3 \leq n \leq 6$. The constraints mentioned are a source of uncertainty for the values n computed with its aid since for certain metals, gold, for instance, the quantity n can be less than one [15] for $\varepsilon_{\lambda,n} \simeq 0.05$.

Broader possibilities for the determination of the optical constants appear if the dependence $D_{\lambda,\theta} = f(\varepsilon_{\lambda,n})$ is used, which is linear for given θ and n. The approximation $D_{\lambda,\theta}$ in the functions $\varepsilon_{\lambda,n}$ and θ has a more complex form

$$D^*_{\lambda,0} = \varepsilon_{\lambda,n}[\exp(a\theta + b)/n + c\theta + d] + \cos\theta, \tag{7}$$

where θ is the emission angle in radians, and a, b, c, d are coefficients dependent on θ: for $\theta = 75°$ $a = 4.86$; b $= -6.59$; c $= 0.477$; d $= -0.245$; for $\theta = 82-84°$ $a = 8.60$; b $= -11.775$; c $= 0.525$; d $= -0.318$.
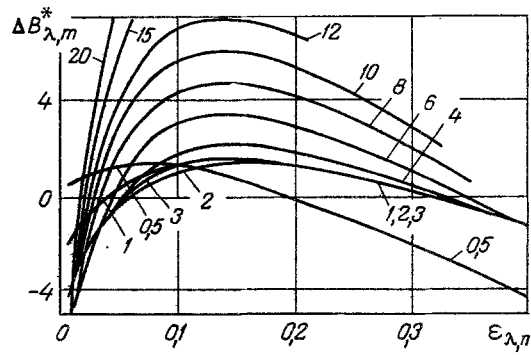
Fig. 2. Error in approximating the dependence $B_{\lambda,m} = f(n, \varepsilon_{\lambda,n})$. Numbers at the curves signify values of the refractive index n. $B^*_{\lambda,m}$, %.

It follows from (7) that

$$n = [\varepsilon_{\lambda,n} \exp(a\theta + b)]/[D_{\lambda,\theta} - \varepsilon_{\lambda,n}(c\theta + d) - \cos\theta]. \qquad (8)$$

Therefore, $\varepsilon_{\lambda,n}$, θ and $B_{\lambda,\theta}$ must be measured to determine the value of n by using (8). In conformity with the rules for calculating errors, we obtain from (8)

$$\Delta n/n = E(\Delta\varepsilon_{\lambda,n}/\varepsilon_{\lambda,n}) - F(\Delta D_{\lambda,\theta}/D_{\lambda,\theta}), \qquad (9)$$

where

$$E = (D_{\lambda,\theta} - \cos\theta)/[D_{\lambda,\theta} - \varepsilon_{\lambda,n}(c\theta + d) - \cos\theta];$$

$$F = D_{\lambda,\theta}/[D_{\lambda,\theta} - \varepsilon_{\lambda,n}(c\theta + d) - \cos\theta].$$

Values of the coefficients E and F, as well as the error in the approximation of the quantity $D_{\lambda,\theta}$ for $\theta = 75°$ and $\theta = 84°$, are presented in Table 1.

It is seen from the data of the table that as the angle increases the accuracy grows in determining n with the error in the approximation taken into account and for an unchanged error in measurement of both $\varepsilon_{\lambda,n}$ and $D_{\lambda,\theta}$. However, not only the error in measuring the relative spectral brightness of the surface, but also the error in setting up and measuring the angle θ determine the magnitude of the error in $D_{\lambda,\theta}$. The diagram presented in Fig. 1, computed by means of (1)-(4), shows the quantity $B_{\lambda,m} = (B_{\lambda,\theta})_{max}$ for different values of $\varepsilon_{\lambda,n}$ and n. It is seen from the diagram that for large values of n and k, corresponding to small $\varepsilon_{\lambda,n}$, $B_{\lambda,m}$ is quite sensitive to the value of the emission angle. Therefore, it is expedient to use a method which does not require measurement of the angle θ to determine the optical constants of metals more exactly in the IR spectrum band.

One such method can be realized if an approximation of the dependence $B_{\lambda,m} = f(n, \varepsilon_{\lambda,n})$ is used, from which there follows

$$n = (1.103\varepsilon_{\lambda,n}^{0.508} B_{\lambda,m})^{\exp(1.327\varepsilon_{\lambda,n}+0.807)}. \qquad (10)$$

The error of the approximation, computed by means of (6) for different $\varepsilon_{\lambda,n}$ and n, is shown in Fig. 2. The error in n, due to the error in measurement of $\varepsilon_{\lambda,n}$ and $B_{\lambda,m}$, is determined by the relationship

$$\Delta n/n = N(\Delta\varepsilon_{\lambda,n}/\varepsilon_{\lambda,n}) + M(\Delta B_{\lambda,m}/B_{\lambda,m}), \qquad (11)$$

where $M = \exp(1.327\varepsilon_{\lambda,n} + 0.807)$; $N = M[1.327\varepsilon_{\lambda,n} \ln(1.103\varepsilon_{\lambda,n}^{0.508}B_{\lambda,m}) + 0.508]$. Values of the coefficients N and M are presented in Table 2.

The approximate value n* which is used to determine the appropriate error in the approximation $\Delta B^*_{\lambda,m}$, by means of Fig. 2, is found from the results of measuring $\varepsilon_{\lambda,n}$ and $B_{\lambda,m}$ by using the diagram of Fig. 1. The value of $B_{\lambda,m}$ corrected by the magnitude of the approximation error is substituted together with $\varepsilon_{\lambda,n}$ in (10) and n is computed.

The data in Fig. 1 and Table 2 show that as $\varepsilon_{\lambda,n}$ grows the accuracy in determining n by the method of measuring $B_{\lambda,m}$ diminishes. Hence, for materials with $\varepsilon_{\lambda,n} > 0.3$ it is more
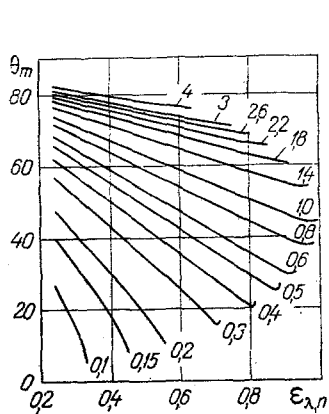
Fig. 3   Fig. 4

Fig. 3. Diagram of the dependence $\theta_m = \varphi(n, \varepsilon_{\lambda,n})$. Numbers at the curves signify values of the refractive index n.
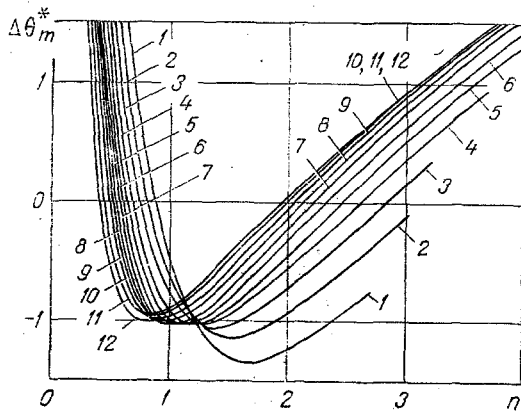
Fig. 4. Error in approximating the dependence $\theta_m = \varphi(n, \varepsilon_{\lambda,n})$: $\varepsilon_{\lambda,n} = 0.25$ (12); 0.30 (11); 0.35 (10); 0.40 (9); 0.45 (8); 0.50 (7); 0.55 (6); 0.60 (5); 0.65 (4); 0.70 (3); 0.75 (2); 0.80 (1), $\Delta\theta_m^* \%$.

expedient to measure the quantity θ_m (the principal angle of incidence [3] or the first Brewster angle [8]) to determine n.

The quantity n which satisfies the equality

$$\theta_m = n \left[\exp(4,42 - 0,96\ln n) - \varepsilon_{\lambda,n}\exp(3.71 - 1.63\ln n)\right] \tag{12}$$

can be found from the results of measurements of θ_m and $\varepsilon_{\lambda,n}$. To do this, the value of $\theta_{m,exp}$ obtained in the measurements should be corrected, as in the previous case, for the magnitude of the approximation error. To do this, the approximate value of n* is determined by means of $\varepsilon_{\lambda,n}$ and $\theta_{m,exp}$ by using the diagram in Fig. 3, computed by means of (2) and (4), and the corresponding magnitude of the approximation error $\Delta\theta_m^*$ is found from the diagram in Fig. 4. Substituting $\theta_m = \theta_{m,exp} + \theta_{m,exp}\Delta\theta_m^*/100$ and n* in (12) and changing n* successively by the magnitude of the established step, the exact value of n whose error due to the error in measuring $\varepsilon_{\lambda,n}$ and θ_m is

$$\Delta n/n = P(\Delta\varepsilon_{\lambda,n}/\varepsilon_{\lambda,n}) + Q(\Delta\theta_m/\theta_m) \tag{13}$$

can be determined by successive approximations. Values of the coefficient Q are presented in Table 3. Computations showed that P $\leq$ 1.5 in the whole range of tabulated values of $\varepsilon_{\lambda,n}$ and n.

The magnitude of the absorption index k and its error for known $\varepsilon_{\lambda,n}$ and n were computed by means of the formulas

$$k = [(4n/\varepsilon_{\lambda,n}) - (n+1)^2]^{1/2} \tag{14}$$

and

TABLE 2. Values of the Coefficients N and M in (11)

n	$\varepsilon_{\lambda,n}$				
	0,05	0,10	0,20	0,30	0,40
0,5	1,2	1,2	1,4	1,5	1,7
1	1,2	1,3	1,5	1,7	1,9
2	1,2	1,4	1,6	1,9	2,2
4	1,3	1,4	1,8	2,1	2,5
6	1,3	1,5	1,8	2,2	2,6
10	1,3	1,5	1,9	2,4	—
15	1,4	1,6	2,0	—	—
M	2,4	2,6	2,9	3,3	3,8

TABLE 3. Values of the Coefficients Q in (13)

n	$\varepsilon_{\lambda,n}$						
	0,25	0,35	0,45	0,55	0,65	0,75	0,85
0,2	2,4	1,4	0,8	0,4	—	—	—
0,4	4,2	2,8	2,0	1,4	1,0	0,7	—
0,6	5,6	3,9	2,9	2,2	1,7	1,3	1,0
0,8	6,6	4,8	3,7	2,9	2,3	1,8	1,5
1,0	7,5	5,6	4,3	3,5	2,8	2,3	1,9
1,4	8,9	6,8	5,4	4,5	3,7	3,1	2,7
2,6	12	9,4	7,8	6,7	5,7	5,0	—
4,0	14	11	10	9	7,3	—	—

TABLE 4. Values of the Optical Constants of Polycrystalline Tungsten at T ∿ 200°C Obtained by Different Methods

$\lambda,$ µm	$\varepsilon_{\lambda,n}$ [15]	$(L_{\lambda,\theta}/L_{\lambda,n})_{max}$	This paper		[15]	
			n	k	n	k
3,5	0,052	3,55	3,8	16,4	3,7	16
5	0,040	4,56	4,8	21,1	5,5	22
6	0,037	5,24	6,0	24,5	6,8	25
7	0,035	6,02	7,7	28,4	8,4	29
8	0,031	6,62	8,4	31,5	10	33
9	0,030	7,12	9,7	34,3	11	35
10	0,029	7,75	11,4	37,7	13	39

TABLE 5. Values of the Coefficient F from (9) and Optical Constants of Polycrystalline Tungsten at T ∿ 200°C for λ = 5 µm ($\varepsilon_{\lambda,n}$ = 0.040). Computed from the Results of Measuring the Relative Spectral Brightness

$\theta,$ deg	$L_{\lambda,\theta}/L_{\lambda,n}$	$A_{\lambda,\theta}$	$D_{\lambda,\theta}$	n	F
75	1,90	0,2625	0,2827	3,66	32,5
82	3,00	0,1415	0,1707	4,80	12,0
83	3,26	0,1239	0,1563	4,73	9,0
84	3,59	0,1063	0,1414	4,89	7,5

$$\Delta k/k = A\,(\Delta n/n) - B\,(\Delta\varepsilon_{\lambda,n}/\varepsilon_{\lambda,n}), \tag{15}$$

where $B = 2(n/k)^2/n\varepsilon_{\lambda,n}$; $A = B[2 - \varepsilon_{\lambda,n}(n + 1)]/2$.

Optical constants of polycrystalline tungsten at T ∿ 200°C, obtained from the results of measuring the maximum relative angular spectral brightness of the natural emission $(L_{\lambda,\theta}/L_{\lambda,n})_{max}$ are presented in Table 4. The data on the spectral normal emissivity of tungsten are taken from [15]. The computation was performed by means of (10) taking account of the approximation error. Since $\theta_m > 86°$ in the whole spectral range of measurement, it was assumed in the computations that $A_{\lambda,\theta} = 0$, $B_{\lambda,m} = 2(L_{\lambda,\theta}/L_{\lambda,n})_{max}$ and the error in $B_{\lambda,m}$, including the error in measurement, was taken equal to 2.5%. Therefore, the error in the values of n due to the error in $B_{\lambda,m}$ according to (11) was ∿6%. Values of the optical constants of analogous tungsten specimens for T = 200°C, obtained by using the Kramers–Kronig method [15], which in conformity with their systematic error exceed the results of this paper by 11% on the average for n and 4% for k, are also presented in Table 4.

Values of the refractive index, computed by using (8) for the same tungsten specimen by means of the results of measuring the relative spectral brightness $(L_{\lambda,\theta}/L_{\lambda,n})$ for several emission angles θ, as well as appropriate values of the coefficient F from (9) are presented in Table 5. The quantity $A_{\lambda,\theta}$ was computed by means of (5) and was used to calculate the value $D_{\lambda,\theta} = 1/[2(L_{\lambda,\theta}/L_{\lambda,n}) - A_{\lambda,\theta}]$. The error in measuring $(L_{\lambda,\theta}/L_{\lambda,n})$ and, therefore, the error in determining $D_{\lambda,\theta}$ did not exceed 1%.

It is seen from Tables 4 and 5 that convergence of the values of n is good for the angles 82-84°, while the value of n for $\theta = 75°$ differs from n = 4.8 by 24%. In conformity with (9) and F = 32.5 we obtain that such a discrepancy is due to the error in determining $D_{\lambda,\theta}$, which equals 0.73%. The value of $D_{\lambda,\theta}$ corrected by this value of the error turns out to be 0.2806. Substituting it into (8), we find n = 4.795.

Therefore, the results obtained show that the approximate expressions proposed in this paper can underlie measurement methods to determine the optical constants of absorbing materials. These methods, being distinguished by simplicity, permit the realization of a complex investigation of the optical and emissive (reflective) properties, and the utilization of existing experimental data on the emissivity and reflectivity of materials [16] to determine the optical constants.

NOTATION

n, k, refractive and absorptive indices ($\hat{n} = n - ik$; $\hat{n}$ is the complex refractive index); α_λ, ε_λ, ρ_λ, spectral absorptivity, emissivity, and reflectivity, respectively; $L_{\lambda,n}$, $\varepsilon_{\lambda,n}$, spectral surface brightness and its normal emissivity; $L_{\lambda,\theta}$, $\varepsilon_{\lambda,\theta}$, spectral surface brightness and its emissivity at an angle θ to its normal; s, p, symbols denoting the composite emissivity (reflectivity) or brightness for emission polarized in parallel (p) and perpendicular (s) to the emission (reflection) plane; θ_m, first Brewster angle; G, exact value of the quantity being approximated, and G* is its approximation; $\Delta G*$, error in the approximation; $A_{\lambda,\theta} = \varepsilon^s_{\lambda,\theta}/\varepsilon_{\lambda,n}$; $B_{\lambda,\theta} = \varepsilon^p_{\lambda,\theta}/\varepsilon_{\lambda,n} = L^P_{\lambda,\theta}/L^P_{\lambda,n} = 1/D_{\lambda,\theta}$.

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